LOYOLA COLLEGE (AUTONOMOUS), CHENNAI – 600 034

B.Sc. DEGREE EXAMINATION – MATHEMATICS

THIRD SEMESTER – APRIL 2010

MT 3501/MT 3500 - ALGEBRA, CALCULUS AND VECTOR ANALYSIS

Date & Time: 23/04/2010 / 1:00 - 4:00 Dept. No. Max. : 100 Marks

PART – A

Answer ALL questions **(10 $\times$ 2 = 20 marks)**

1. Evaluate $\int_0^a\int_0^b\int_0^c (x+y+z)\,dxdydz$
2. Show that β(m, n) = β(n, m).
3. Solve $x + y\frac{\partial z}{\partial x} = 0$.
4. Solve $p = y^2 q^2$.
5. Show that the vector $\bar{F} = 3y^4z^2\vec{i} + 4x^3z^2\vec{j} - 3x^2y^2\vec{k}$ is solenoidal.
6. State Stoke's theorem.
7. Find $L\left(te^{-at}\right)$.
8. Find $L^{-1}\left(\frac{1}{s(s+a)}\right)$
9. What is the highest power of *2* in *79*!?
10. State Fermat's theorem.

PART – B

Answer any FIVE questions **(5 $\times$ 8 = 40 marks)**

11. Change the order of integration and hence evaluate $\int_{-a}^{a}\int_{0}^{\sqrt{a^2-y^2}} xdxdy$.

12. Using Gamma function evaluate $\int_0^\infty e^{-x^2}dx$.

13. Solve $x^2p^2 + y^2q^2 = z^2$.

14. Solve $y^2p + x^2q = x^2y^2z^2$.

15. Find (i) $L\left(te^{-t}\sin t\right)$ (ii) $L\left(\frac{\sin at}{t}\right)$

(P.T.O.)

16. Find $L^{-1}\left(\frac{s+2}{\left(s^2+4s+5\right)^5}\right)$

17. If $\bar{F} = xy^2\vec{i} + 2x^2yz\vec{j} - 3yz^2\vec{k}$, find $div\,\bar{F}$ and $curl\,\bar{F}$. What are these values at (1, -1, 1)?

18. Show that every integer which is a perfect cube is of the form $7p$ or $7p \pm 1$.

PART – C

Answer any TWO questions **($2 \times 20 = 40$ marks)**

19. (a) Evaluate $\iint_R (x-y)^4 e^{x+y}\,dxdy$ where R is the square with vertices (1, 0), (2, 1), (1, 2) and (0, 1).

(b) Using β and Γ-functions evaluate (i) $\int_0^1 x^7(1-x)^3 dx$ (ii) $\int_0^{\pi/2} \sin^5 x \cos^3 x\, dx$.

20. Solve: (a) $z^2 (p^2 + q^2 + 1) = b^2$. (b) $(x^2 - yz)p + (y^2 - zx)q = z^2 - xy$.

21. (a) Verify Green's theorem in the plane for $\int_C (3x^2 - 8y^2)dx + (4y - 6xy)dy$ where C is the boundary of the region defined by $y = \sqrt{x}$ and $y = x^2$.

(b) Show that if x and y are both prime to the number n, then $x^{n-1} - y^{n-1}$ is divisible by n. Deduce that $x^{12} - y^{12}$ is divisible by 1365.

22. (a) Using Laplace transform solve $\frac{d^2y}{dx^2} + 6\frac{dy}{dx} + 5y = e^{-2t}$ given that y = 0; $\frac{dy}{dx} = 1$ when t = 0.

(b) Prove that the 5th power of any integer N has the same units digit as N.

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